

*Solution of a Time-Dependent, One-Dimensional Neutron Transport Problem**

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1. The fundamental equation. In [1] the following simple model of neutron transport is considered: Let there be a rod of length x made of fissionable material. A neutron moving through the rod at constant speed c has a probability $\lambda^{-1}y + o(y)$ of causing a fission in a distance y . A fission results in the disappearance of the original neutron and the emergence of two similar neutrons, one going to the left, the other to the right. We ask: If a neutron is introduced into one end of a rod of length x at time $t = 0$, what is the expected number $U(x, t)$ of neutrons emerging from that end in the time interval $[0, t]$?

The equation for $U(x, t)$ is derived in [1]. We find

$$(1.1a) \quad \frac{\partial U}{\partial x} + \frac{2}{c} \frac{\partial U}{\partial t} = \lambda^{-1} \int_0^t u(x, z) U(x, t - z) dz + \lambda^{-1}, \quad x \geq 0, t \geq 0,$$

where

$$(1.1b) \quad u(x, t) = \frac{\partial U}{\partial t}(x, t).$$

The function $U(x, t)$ must satisfy the conditions

$$(1.1c) \quad U(x, 0) = 0, \quad U(0, t) = 0.$$

It is convenient to change both time and distance scales by the transformation

$$(1.2) \quad x' = \lambda^{-1}x, \quad t' = c\lambda^{-1}t.$$

Then, on suppressing the primes and using the same functional notation, we get

$$(1.3) \quad \frac{\partial U}{\partial x} + 2 \frac{\partial U}{\partial t} = 1 + \int_0^t u(x, z) U(x, t - z) dz, \quad U(x, 0) = 0, \quad U(0, t) = 0.$$

We shall further assume that $U(x, t)$ is sufficiently well behaved to differentiate (1.3) to obtain

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