

# *On an estimate for the curvature of minimal surfaces $z = z(x, y)^*$*

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Let the function  $z = z(x, y)$  be of class  $C^2$  and a solution of the minimal surface equation in the disc  $x^2 + y^2 < R^2$ . Introduce the isothermic parameters

$$(1) \quad \begin{aligned} \xi &= x + \int_{(0,0)}^{(x,y)} W^{-1}[(1 + p^2) dx + pq dy] \equiv x + A(x, y), \\ \eta &= y + \int_{(0,0)}^{(x,y)} W^{-1}[pq dx + (1 + q^2) dy] \equiv y + B(x, y). \end{aligned}$$

Here  $W = (1 + p^2 + q^2)^{1/2}$ . Put  $\xi + i\eta = \zeta = \zeta(x, y)$  and  $\rho = \min |\zeta|$  for  $x^2 + y^2 = R_\epsilon^2$  ( $R_\epsilon = R - \epsilon$ ). Without loss of generality for the following it may be assumed that  $|\zeta(R_\epsilon, 0)| = \rho$ .<sup>1</sup> It has been shown in [6] that the mapping  $x, y \rightarrow \zeta$  enlarges distances. Hence  $\rho > R_\epsilon$ . The function

$$(2) \quad F(\zeta) = \frac{p - iq}{1 + W} = \frac{d}{d\zeta} \left[ z(x, y) + i \int_{(0,0)}^{(x,y)} W^{-1}(p dy - q dx) \right]$$

is a regular-analytic function of  $\zeta$ ,<sup>2</sup> and  $|F(\zeta)| < 1$ . For the Gaussian curvature  $K$  one finds ([6], p. 418)

$$(3) \quad K = -|F'|^2 \left( 1 + \frac{1}{W} \right)^4.$$

The application, as in [4], of PICK's lemma (see [2], p. 14) yields

$$(4) \quad |F'(0)| \leq \frac{1}{\rho} (1 - |F(0)|^2) = \frac{2}{\rho(1 + W_0)}.$$

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<sup>1</sup> A rotation in the  $x, y$ -plane results in the same rotation in the  $\zeta$ -plane which does not change distances.

<sup>2</sup> See for instance L. BERS [1], p. 367; [6].