

Boundary Estimates for Second Order Parabolic Equations and Their Applications

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Introduction. In [5] we derived interior Schauder type estimates for parabolic systems of any order. Those estimates have been used to prove differentiability theorems for both linear and nonlinear parabolic systems. The problem of deriving analogous estimates near the boundary, for general parabolic systems, is still open. However, for second order parabolic equations, these estimates have been derived by RICHARD B. BARRAR (in his unpublished thesis [0]) and later on, for a one-dimensional space variable, by CILIBERTO [1]. In this paper we shall derive these estimates by a simpler method suggested from the analogous elliptic case which was considered in [2] by DOUGLIS & NIRENBERG. Our derivation might serve as a model to derive the analogous boundary estimates for general parabolic systems.

There is one essential difficulty which arises in the parabolic case (for boundary estimates) and which does not arise in the analogous elliptic case. This is explained in the proof of Theorem 1 (Section 2). To overcome that difficulty we prove (in Section 1) Lemma 1. Theorem 1 is analogous to Theorem 1 in [5]. In Section 3 we prove the desired estimates, via Theorem 1. The proof is similar to that of [5]. Finally, in Sections 4 and 5 we give applications of the boundary estimates by deriving existence theorems for linear and some nonlinear parabolic equations. Our method involves a well known continuity argument (see, for instance, SCHAUDER [13], NIRENBERG [11]). Theorem 4 was also proved by R. B. BARRAR by the continuity argument, though in other respects his proof differs from our proof.

1. A FUNDAMENTAL LEMMA

Denote by R the interval $-1 < x < 1$, by S_τ ($S_1=S$) the rectangle $-1 < x < 1$, $0 < t < \tau$, and by ∂S_τ the *normal* boundary of S_τ , that

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