

On Quasi-Linear Parabolic Equations of the Second Order

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Introduction. Let D denote a bounded $(n + 1)$ -dimensional domain and let its points be denoted by $(x, t) = (x_1, \dots, x_n, t)$. Let D be bounded by two hyperplanes $t = 0$ and $t = t_0 > 0$, and by a surface C (C is a closed set) lying between these hyperplanes. Denote by \bar{D} the closure of D , by B the basis of D ($B = \bar{D} \cap \{t = 0\}$), by D_τ the subdomain $D \cap \{0 < t < \tau\}$, by C_τ the intersection $C \cap \{0 \leq t \leq \tau\}$ and, finally, denote by ∂D_τ (∂D) the set $C_\tau \cup B$ ($C \cup B$). Consider the linear operator

$$(1) \quad Lu \equiv \sum_{i,j=1}^n a_{ij}(x, t) \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^n b_i(x, t) \frac{\partial u}{\partial x_i} + c(x, t)u - \frac{\partial u}{\partial t}$$

with coefficients defined in \bar{D} . In Section 1 and occasionally in Section 2 we shall assume that the coefficients of L satisfy the following assumptions:

(A) *The functions a_{ij} have continuous derivatives up to the fifth order with respect to (x, t) in \bar{D} ; the functions b_i and c have continuous derivatives up to the third order with respect to (x, t) in \bar{D} .*

(B) *L is of parabolic type, that is, there exists a positive constant H such that for every point (x, t) in \bar{D} and for every real vector $\xi = (\xi_1, \dots, \xi_n)$*

$$\sum_{i,j=1}^n a_{ij}(x, t)\xi_i\xi_j \geq H \sum_{i=1}^n \xi_i^2.$$

Throughout Section 1 and occasionally in Section 2 we shall assume that the surface C satisfies the following property:

(E') *To every point P of C there exists an $(n + 1)$ -dimensional sphere V with center P such that the intersection $C \cap V$ can be represented, for some i ($1 \leq i \leq n$), in the form*

$$(2) \quad x_i = h(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n, t)$$

where h has continuous derivatives up to the sixth order in all its arguments.

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