

The Kernel-Function and Conformal Invariants

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1. Introduction. In this paper we present some applications to the theory of conformal mapping of certain invariants which are closely related to the Bergman kernel-function. In particular, it will be shown that such invariants, which can be expressed in terms of the solutions to certain extremal problems of a rather simple type, may be used to determine the conformal modulus of a doubly-connected domain. It will be readily apparent that the methods employed here may be used to define an unlimited number of invariants which could also be used in similar investigations. It appears likely that Theorems 1 and 2, which deal with doubly-connected domains, can be generalized in such a manner as to relate some of these invariants to the conformal moduli of domains of higher connectivity. We hope to return to such problems in a later paper.

2. The Bergman Kernel-function of a Plane Domain. In this section we present briefly, and without any proofs, the most essential facts concerning the kernel-function of a plane domain. For a detailed exposition the book [1] of BERGMAN and his original papers may be consulted.

Let a domain D be given in the plane of the complex variable $z = x + iy$, and let H_D be the family of all functions which are analytic, single-valued, and quadratically integrable in D , *i.e.*,

$$(1) \quad \iint_D |f(z)|^2 dx dy < \infty.$$

If the inner product is defined in the usual manner,

$$(2) \quad (f, g) = \iint_D f(z) \overline{g(z)} dx dy,$$

H_D becomes a (separable) Hilbert space. (We assume that the boundary of D

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