A Class of Quasi-Pseudo-Conformal Transformations in the Theory of Functions of Two Complex Variables*

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1. A class of quasi-pseudo-conformal transformations. When developing the theory of functions of two complex variables we are interested in following, at least to a certain degree, the pattern used in the theory of functions of one complex variable. The possibility of solving the Dirichlet problem for harmonic functions of two real variables is a powerful tool in the theory of conformal mapping. The class of B-harmonic functions (real parts of analytic functions of two complex variables) is too small to allow the solution of the boundary value problem, even for domains with a distinguished boundary surface and with values prescribed only on this surface. Therefore it is indicated (e.g. when generalizing the Nevanlinna approach to the theory of two complex variables) that one should introduce functions of some suitable extended class. One can define, in the case of domains with a distinguished boundary surface, various extended classes of functions. See Bergman [3, 4, 8, 10], Bremermann [13, 14], Lowdenslager [19, 20].

It is of considerable interest to introduce extended classes of functions for other types of domains in the space of two complex variables, and to study mappings defined by pairs of functions of a certain extended class.

A transformation of a domain in the space of two complex variables by a pair $\{Z_k(z_1, z_2)\}$, k = 1, 2, of analytic functions of z_1 , z_2 with a non-vanishing and finite functional determinant is denoted as a PC (pseudo-conformal) transformation.

As is well known, domains of the space of two complex variables cannot in general be transformed into each other by PC transformations. For many purposes it is therefore useful to introduce more general transformations which then permit us to map into each other larger classes of domains. Obviously, mappings of this type can be defined in various ways. In particular, it is of

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