

Some Compact Solvmanifolds and Locally Affine Spaces

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INTRODUCTION

Much of the early algebraic work in the study of Lie groups was centered about the classification of the simple Lie groups and related topics. In contradistinction to this, MALCEV in [5] initiated a study of nilpotent Lie groups. This work deals mainly with the existence of closed subgroups of nilpotent Lie groups such that the homogeneous space is compact. (We will often refer to the homogeneous space of a nilpotent Lie group as a nilmanifold.) More explicitly, MALCEV in [5] provides solutions to the following three problems:

Problem 1. Given a connected, simply connected, nilpotent Lie group N , when does there exist a closed subgroup Γ such that N/Γ is compact?

Problem 2. Give an abstract group theoretic characterization of the fundamental groups of compact nilmanifolds.

Problem 3. Given two compact nilmanifolds with isomorphic fundamental groups, are the manifolds homeomorphic?

There has been some work in recent years [4], [6] trying to extend MALCEV's results to compact homogeneous spaces of solvable Lie groups or, more briefly, solvmanifolds. In [6], MOSROW showed that the answer to Problem 3 is in the affirmative for compact solvmanifolds. In [4], WANG gave a necessary and sufficient condition for a group Γ to be the fundamental group of a solvmanifold (not necessarily compact). It is the purpose of this paper to show that a certain class of groups Γ consists of fundamental groups of *compact* solvmanifolds. In other words, we give a partial solution of Problem 2 for the case of compact solvmanifolds.

In order to simplify the presentation of the material in this paper, we will adopt the following conventions: Z^s will denote the direct sum of the additive group of integers taken s times, R will denote the real field and C will denote the complex field. The symbols $Z^t \rightarrow Z^t \otimes R$ or $Z^t \rightarrow Z^t \otimes C$ will denote the