

Representation of a Markovian Semigroup and Its Infinitesimal Generator

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Consider an order preserving semigroup of linear transformations P^t on a space of functions. Such semigroups are of interest in the theory of Markoff processes. It is well known that the Markoff process associated with such a semigroup possesses many desirable regularity features in case the space of functions is a $\mathcal{C}(X)$ (all continuous functions vanishing at infinity on a locally compact Hausdorff space X) and the semigroup P^t is strongly continuous. In Section 1 such a semigroup acting on an algebra of functions is represented as acting on a $\mathcal{C}(X)$. In general, the semigroup is not strongly continuous. There is a maximal linear subspace \mathcal{U} of $\mathcal{C}(X)$ on which P^t is strongly continuous. The set \mathcal{A} of all f such that f and f^2 are both in \mathcal{U} is shown to be an algebra, but an example shows that \mathcal{A} need not be invariant under the action of P^t . Thus the desired situation that P^t be strongly continuous on an algebra of functions corresponds to a regularity assumption of an algebraic nature.

Section 2 considers the problem of representing the infinitesimal generator in terms of a family of linear functionals defined at each point. Such a representation (somewhat similar to that of HUNT [3] in the spatially homogeneous case) is established, but it holds only for functions in the multiplier algebra \mathfrak{M} of the domain \mathfrak{D} of the infinitesimal generator. An example is given to show that \mathfrak{M} may reduce to the constant functions alone, and that such a representation on \mathfrak{D} itself is not always possible. Again, the representation is only of interest when a regularity condition of an algebraic nature is satisfied.

1. Representations and strong continuity. Let \mathcal{C} be a real commutative C^* algebra. Examples are all bounded measurable functions on a measurable space, \mathcal{L}^∞ on a measure space, or the set $\mathcal{C}(X)$ of all continuous functions vanishing at infinity on a locally compact Hausdorff space X . The norm in these examples is the supremum norm (or essential supremum norm in the case of \mathcal{L}^∞). Any

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