

# *On Quasi-Conformal Functions of Several Complex Variables\**

SIN HITOTUMATU

*Communicated by S. BERGMAN*

## 1. Introduction.

1.1. The well known Riemann mapping theorem asserts that every simply connected plane region with at least two boundary points can be mapped onto the unit disc  $\{|z| < 1\}$  conformally, *i.e.*, by a holomorphic transformation. However, a similar situation does not hold in the case of several complex variables. As POINCARÉ [12] showed in 1907, the bicylinder  $\{|z_1| < 1, |z_2| < 1\}$  and the hypersphere  $\{|z_1|^2 + |z_2|^2 < 1\}$  cannot be mapped onto each other by a one-to-one holomorphic transformation of two complex variables, although these domains have the same structure from the purely topological point of view. In reality, the holomorphic rigidity in the case of several complex variables is far more severe than in the case of one complex variable.

Of course, even in the case of one complex variable, multiply connected domains or domains on Riemann surfaces are not always conformally equivalent to each other, even when they have the same topological structure. For example, the ring domain  $\{q < |z| < 1\}$  is a representative domain of the class of doubly connected domains, and two ring domains  $\{q < |z| < 1\}$  and  $\{q' < |w| < 1\}$  are conformally equivalent to each other if and only if  $q = q'$ . Hence  $q$  is a conformal invariant. For a doubly connected domain  $D$  which is mapped conformally onto  $\{q < |z| < 1\}$ , the quantity

$$(1) \quad \text{mod } D = \frac{1}{2\pi} \log \frac{1}{q}$$

is known as the *modulus* of the domain  $D$ .

1.2. The theory of quasi-conformal mappings of one complex variable has recently been extensively developed (for instance, [1], [6], [9], [11], [15], [16]). First I shall give a brief sketch of the main ideas of this theory, because it is necessary for the later discussion.

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