

Spectral Representation of Certain Semi-Groups of Operators

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Summary. In section 1 some definitions and results are given which will be used in the rest of the paper. The main results are proved in sections 4 and 7. In section 2 the characters of a certain class of semi-groups are studied. Section 5 deals with semi-groups of bounded scalar operators. In sections 3 and 6 a series of propositions on hermitian spectral families are given.¹

1. The semi-group algebra. Representations. Let G be a locally compact group, S a locally compact subspace of G and m the restriction of a left invariant Haar measure to S . Suppose

- (1) $st \in S$ if $s \in S, t \in S$,
- (2) $m(U) > 0$ for every non-void set $U \subset S$, open in S .

Let $M(S)$ be the vector space of complex Radon measures defined on S , with compact support; for every measure μ let $S(\mu)$ be its support. For $\mu \in M(S)$, $\nu \in M(S)$ we can define the product $\mu\nu$ by means of the relations

$$\int f(s) d\mu\nu(s) = \iint f(st) d\mu(s) d\nu(t) \quad \text{for } f \in \mathcal{R}(S);$$

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