

On Vitali's Theorem for Complex Spaces with Singularities

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Communicated by S. S. CHERN

The subject of complex spaces with singularities has been the scene of a great deal of recent activity in the theory of functions of several complex variables; and several function-theoretic results have been extended from complex manifolds to these more general spaces. The maximum modulus and Vitali convergence theorems have been treated in [3], for example; but there use was made of a rather explicit local representation of the analytic spaces as analytic subvarieties. The present note intends to furnish a simpler proof of these extensions, which will be valid in considerably more general circumstances, and which will be self-contained in so far as properties of spaces with singularities are concerned. It will be seen in this approach that these theorems follow from the corresponding theorems for complex manifolds as essentially topological rather than analytical consequences. As an application, it will be demonstrated that the full group of analytic automorphisms of a compact complex space with singularities is a Lie group.

The first section is devoted to the proof of the maximum modulus and convergence theorems for a large class of branched coverings of complex manifolds; the approach taken here involves a rather more extensive use of the order of branching than customary and a definition of holomorphic functions which is particularly suited to the theorems to be proved. In the second section this definition of holomorphic functions is re-examined on a restricted class of branched coverings, for which the definition can be put in a perhaps more natural form. The relationship between this restricted class of branched coverings and the branched coverings introduced by BEHNKE & STEIN [1] is discussed in the third section. The last section is devoted to the demonstration of the local compactness of the group of analytic automorphisms of a compact analytic space, from which the theorem that that group is a Lie group follows as an immediate consequence of known results.

1. Let V be a separable, connected, Hausdorff space; π be a continuous open mapping of V onto a bounded open subset D of the space of n complex variables;