

# *Composition of Solutions of Linear Partial Differential Equations in Two Independent Variables*

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The purpose of this paper is to establish a formula which associates with any two solutions  $u$  and  $v$  of a linear homogeneous partial differential equation with constant coefficients in two independent variables  $x, y$  a new solution  $w$  which depends bilinearly on  $u$  and  $v$ .

**Notations.** Let  $O$  stand for the origin of the  $x, y$ -plane and  $P$  and  $Q$  for points of a simply connected neighborhood of  $O$ . A function of the point  $Q$ , say  $f(Q)$ , will also be written as  $f(x_Q, y_Q)$  or  $f(\vec{OQ})$  or  $f(\vec{P_1P_2})$ , provided the vector  $\vec{P_1P_2}$  has the same components  $x_Q, y_Q$  as  $\vec{OQ}$ . Differentiation with respect to  $x_Q$  or  $y_Q$  will be denoted by multiplication from the left by  $\xi$  or  $\eta$ , while differentiation with respect to  $x_P$  or  $y_P$  will be denoted by multiplication from the left by  $\xi_P$  or  $\eta_P$ , respectively. It is assumed that all derivatives occurring are continuous unless stated otherwise, and consequently the order of differentiation will not affect the result.

We set

$$Lv(Q) = L_Q v(Q) = \sum a_{kl} \xi^k \eta^l v(x_Q, y_Q)$$

where the sum is over all non-negative integers  $k, l$  with  $k + l \leq \rho$ , and we suppose that not all coefficients of order  $\rho$  vanish. If  $Mv$  is the adjoint differential expression then evidently

$$M_Q u(\vec{QP}) = L_P u(\vec{QP}) = \sum a_{kl} \xi_P^k \eta_P^l u(x_P - x_Q, y_P - y_Q).$$

In Green's formula

$$\iint_D [u(Q)Lv(Q) - v(Q)Mu(Q)] dx dy = \int_{\partial D} B_1(u, v) dx + B_2(u, v) dy$$

the bilinear forms  $B_1(u, v)$  and  $B_2(u, v)$  are not uniquely determined. If  $B'_1(u, v)$

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