

The Solution and Huygens' Principle for a Singular Cauchy Problem

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1. Introduction. The subject of this paper is a study of a singular Cauchy problem for a hyperbolic partial differential equation in $m + 1$ variables. The classical problem in this domain is that for the Euler-Poisson-Darboux (EPD) equation, which in its full generality has been investigated extensively since the appearance of WEINSTEIN's paper in 1952. Our equation [see (1.1)], restricted to two variables, goes back also to EULER and DARBOUX. In 1955, without connection with the earlier studies, STELLMACHER, investigating Huygens' principle, found for (1.1) the fundamental solution and thus, in principle, solved the *regular* Cauchy problem.

The equation which we consider is

$$(1.1) \quad \frac{\partial^2 u}{\partial t^2} + \frac{k}{t} \frac{\partial u}{\partial t} - \sum_{i=1}^m \left(\frac{\partial^2 u}{\partial x_i^2} + \frac{\lambda_i}{x_i^2} u \right) \equiv L(u) = 0$$

where k and λ_i are real or sometimes complex parameters. We exclude the so-called exceptional values, $k = -1, -3, -5, \dots$ [1], [14]. The *singular* Cauchy problem corresponds to the initial data

$$(1.2) \quad u(x, 0) = f(x), \quad u_t(x, 0) = 0$$

in which, for convenience, we write x in place of x_1, x_2, \dots, x_m and use subscripts to denote differentiation. The differentiability assumptions on f will be formulated later. The differential equation in the form considered by STELLMACHER is obtained from (1.1) by putting $u = vt^{-\frac{1}{2}k}$. For $\lambda_i = 0$ for every i (1.1) reduces to the EPD equation. Assuming that for each i $|x_i| > |t| \geq 0$, the right hand sum is a regular analytic elliptic differential operator $X(u)$, and therefore (1.1) belongs to the class of singular hyperbolic equations considered by BUREAU in 1955. For such equations BUREAU used new devices to develop a solution for the singular Cauchy problem. His result generalizes WEINSTEIN's for the EPD equation, but it is valid only in the small and has the character of an existence theorem rather than that of an explicit solution. Some points, such as differentiability conditions, uniqueness, and Huygens' principle, are not examined.