

*Solutions of a Riccati Matrix Differential Equation as Functions of Initial Values**

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1. Introduction. REDHEFFER [1, 2] has recently studied the character of the solution $u = u(x, y)$ of the scalar Riccati differential equation and initial condition

$$(1.1) \quad u_y = a(y) + 2b(y)u + c(y)u^2, \quad u(x, x) = 0,$$

as a function of the two variables (x, y) . If $v = v(x, y)$ and $w = w(x, y)$ are defined in terms of $u(x, y)$ by the equations

$$(1.2) \quad v_y = b(y) + c(y)u(x, y), \quad v(x, x) = 0,$$

$$(1.3) \quad w_y = c(y) \exp \{2v(x, y)\}, \quad w(x, x) = 0,$$

then he showed that, as functions of x , the triple u, v, w satisfies a system of differential equations similar to the above, and also certain non-linear functional equations. Moreover, REDHEFFER proved that if these functional equations are satisfied by a triple of functions u, v, w which are continuous, are such that $u(x, x) = v(x, x) = w(x, x) = 0$, and the partial derivatives u_y, v_y, w_y exist along the line $y = x$, then there exist functions $a(y), b(y), c(y)$ such that u, v, w are solutions of (1.1), (1.2), (1.3).

It is the purpose of the present paper to extend the results of REDHEFFER to matrix differential equations of Riccati type. The method of proof used here is based on the properties of solutions of a related system of linear differential equations. It is felt that this method of attack is simpler than that employed by REDHEFFER for the scalar problem. Moreover, for the case considered by him the present method shows that the $v(x, y)$ in Theorem I of REDHEFFER [1] is continuous in (x, y) , so that the assumption of continuity stated there is extraneous; also, the continuity assumption of Theorem IV of [1] is shown to be unnecessary.

Matrix notation is used throughout the present paper; in particular, matrices

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