

# On Certain Trigonometrical Expansions

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**1. Introduction.** The work described in this paper arises from a study of the long waves in the North Sea generated by a wind field.

In this context there occurs the problem of<sup>1,2</sup> how to determine a solution of the equation of HELMHOLTZ

$$(1.1) \quad \frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} - q^2 F = 0,$$

in the region  $0 < x < \pi$ ,  $y > 0$ , which vanishes for  $y \rightarrow \infty$ , and which satisfies the boundary conditions

$$(1.2) \quad F = 0 \quad \text{at} \quad x = 0 \quad \text{and} \quad x = \pi,$$

$$(1.3) \quad \cos \mu\pi \frac{\partial F}{\partial y} - \sin \mu\pi \frac{\partial F}{\partial x} + f(x) = 0 \quad \text{at} \quad y = 0.$$

It will be assumed that  $q$  and  $\mu$  are given real constants with  $q \geq 0$  and  $0 \leq \mu \leq \frac{1}{2}$ , and that  $f(x)$  is a given real function. In order to simplify the discussion we shall assume that  $f(x)$  is an analytic function. In many cases, however, a much less stringent condition may be sufficient.

The condition (1.3) means that at  $y = 0$  the solution  $F(x, y)$  has a prescribed directional derivative, the direction of which makes a positive angle  $\mu\pi$  with the normal. Physically  $\operatorname{tg} \mu\pi$  is proportional to the coefficient of CORIOLIS, *i.e.* the angular velocity of the earth.

The solution  $F(x, y)$  may be represented as follows

$$(1.4) \quad F(x, y) = \sum_1^{\infty} b_k \frac{\sin kx}{k} e^{-y\sqrt{k^2+q^2}}.$$

The coefficients  $b_k$  can be determined by means of the boundary condition (1.3) at  $y = 0$  which gives

$$(1.5) \quad f(x) = \sum_1^{\infty} b_k \left\{ \sqrt{1 + \frac{q^2}{k^2}} \cos \mu\pi \sin kx + \sin \mu\pi \cos kx \right\}.$$

<sup>1</sup> Cf. G. I. TAYLOR, *Proc. Lond. Math. Soc.* (2) XX 148 (1920).

<sup>2</sup> Cf. H. LAMB, *Hydrodynamics*, 6<sup>th</sup> ed., art. 209 (1932).