

The Derivation of Stress-Deformation Relations for a Stokesian Fluid

JAMES SERRIN

In continuum mechanics the action of material forces across a surface element is shown to depend upon a certain tensor \mathbf{T} , known as the stress tensor. Through its appearance in the equations of motion, this tensor governs the dynamic response of the medium in question; thus by relating \mathbf{T} to other kinematic and thermodynamic variables we effectively define or delimit the type of medium which we study, *e.g.* fluid, elastic, plastic, and so forth. Such a relation between \mathbf{T} and other variables is known as a *constitutive equation*, and its form is obviously of fundamental importance. In the case of fluids, the constitutive equation usually adopted is the classical Newton-Cauchy-Poisson viscosity law, but recently there has been considerable interest in more general stress-deformation relations, [3]–[8], [10]–[12]. The purpose of this paper is to give a simple and complete treatment of the constitutive equation in one of these more general situations, namely when the stress tensor is a function of the deformation tensor (Stokesian fluid). In §1 a clear-cut set of conditions is stated concerning the reaction of the fluid to deformation; from these conditions and these alone the form of the constitutive equation follows unavoidably. The algebraic details are presented in §§ 2 and 4, with the main results being stated in the form of two theorems. In §3 we have included a simple proof of the Newton-Cauchy-Poisson law, which is not so well known as it deserves to be. Finally, in §5 the concept of fluid pressure is discussed and clarified in the light of the preceding work.

Some of the results of the paper are not new. In particular, the main theorems 1 and 2 are essentially due to RIVLIN & ERICKSEN [7], and the discussion of fluid pressure is largely taken from TRUESDELL's memoir [12]. Thus part of the paper is expository; it is hoped, however, that the simplicity of the derivations, the inclusion of some new results, and, above all, the fact that the material is available here for the first time as a unified theory will justify the author's efforts. Finally, we should emphasize that our point of view stems directly from the pioneer work of STOKES, and is greatly influenced by the "phenomenological" ideas of REINER, RIVLIN, and TRUESDELL, who first seriously studied constitutive equations for non-Newtonian fluids.

1. The Stokesian fluid. In a remarkable paper published when he was but 26 years old, STOKES put the concept of fluidity into the following words: "That