

*Free Boundary Problems for Parabolic Equations I. Melting of Solids**

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Communicated by H. LEWY

Introduction. Free boundary problems for the heat equation have been considered for over a century (for references prior to 1929 see BRILLOUIN [1]). In a few special cases the solutions have been found explicitly, but existence theorems for general cases were first proved about ten years ago. The well known one-dimensional Stefan problem, concerning the melting of ice when heated at the boundary with a prescribed temperature, was considered by RUBINSTEIN [22], [23], [24], [25], [26], [27] and DACEV [2], [3], [4], [5] (see also DATZEFF [6]). They used different methods whose basic features can be found in their papers [22] and [2] respectively.

Another Stefan problem where the flow of heat is given at the boundary was considered by EVANS [11] (see also [10]), J. DOUGLAS & GALLIE [7], SESTINI [28], [29], MIRANKER [20], J. DOUGLAS [8] and KYNER [19]. In the last two papers this problem (existence and uniqueness for all future times) was completely solved, regardless of whether both ice and water or just ice alone was present at the beginning.

The papers of DACEV on the Stefan problem are of heuristic nature. Actually he did not give any precise proof of his assertions about existence and uniqueness. It seems that it should be quite involved to rigorize his method. On the other hand, RUBINSTEIN [22] proved existence and uniqueness for small intervals of time except for a few incomplete details. In [25] he stated that he can prove existence and uniqueness for any interval of time with the aid of certain lemmas on "superparabolic" functions.

A method for solving some free boundary problems was developed by KOLODNER [18]. This method was used by MIRANKER [20] to solve a Stefan problem mentioned above. It was earlier applied by KELLER, KOLODNER & RITGER [14], KOLODNER [15], [16] and KOLODNER & RITGER [17] to study problems of evaporation and condensation of liquid drops.

In this work we refine the method of RUBINSTEIN [22] and use it to solve

*Prepared under ONR Contract Nonr-222 (37) (NR 041 157) with the University of California in Berkeley.