

*On Nonlinear Differential Equations,  
the Maximum Operation, and  
Monotone Convergence*

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*Communicated by R. BELLMAN*

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**Summary.** It is shown that the solutions of certain classes of nonlinear ordinary and partial differential equations may be represented in terms of the maximum operation applied to the solutions of associated linear equations. This, in effect, affords a new approach to the quasilinearization of nonlinear differential equations. This technique was first used by BELLMAN, [10], who obtained, in particular,