

# *Invariant Imbedding and Neutron Transport Theory IV—Generalized Transport Theory*

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**1. Introduction.** In a series of recent papers the principle of invariant imbedding has been utilized in the study of a variety of physical processes: radiative transfer [1, 2, 3, 4, 5], neutron transport [6, 7, 8, 9], random walk and scattering [10], and wave propagation [11, 12].

The aim of the present paper is to extend previous results and techniques so as to include an extensive category of transport processes involving deterministic and stochastic interactions and general geometries. Characteristic functions and probabilities as well as expected values (fluxes) are determined.

For intuitive reasons the reader may use neutron transport theory as a model, though we encase the problem in a more abstract setting in order to emphasize the common features of a number of particle processes. In this way we obtain an extensive generalization of the fundamental invariance principles of AMBARZUMIAN [13] and CHANDRASEKHAR [14].

In §§2–3 we present a rather general formulation of the flux or expected value problem. A single equation is developed which automatically accounts for different geometries, energy distributions, *etc.* This formula includes many of the problems considered in past work [1, 2, 6, 7, 8] and leads to new results. Some specializations are indicated in §4.

In §5 a particularly simple problem is investigated from a stochastic viewpoint. The resulting equation is analyzed and it is shown that the flux equation can be obtained from it. The last section deals with a general stochastic analysis. It is indicated that the results of §3 may be obtained by taking moments of the general stochastic equation.

**2. Description of a Generalized Transport Process.** Let a family of surfaces in  $n$ -dimensional space be characterized by a single continuous parameter  $\eta \geq 0$ . The surface corresponding to  $\eta = 0$  (it may be a degenerate surface) will be considered a bounding surface, and the parametrization will be such that the region included between  $\eta = 0$  and  $\eta = \eta_1$  will be included by the region

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