

The Holonomy Group

II. The Lie Group Induced by a Tensor*

V. HLAVATÝ

Summary. This paper is a continuation of [2]. It deals with the Lie group induced by an arbitrary tensor. The first part describes the construction of the induced Lie group. The second part deals with the application to the curvature tensor. The Lie group induced by the curvature tensor is the holonomy group. The holonomy group is a very convenient tool for solving partial differential equations for covariant constant tensors.** The third part introduces some basic notions and auxiliary results which will be used in the next paper of this series.***

I. THE LIE GROUP INDUCED BY A TENSOR

1. The basic notion. Denote by L_n an n -dimensional space endowed with a symmetric connection $\Gamma_{\lambda}^{\nu\mu}$ and by ∇_{μ} the symbol of the covariant derivative with respect to $\Gamma_{\lambda}^{\nu\mu}$. The local tangential flat space to L_n at a generic point P will be denoted by A_n or $A_n(P)$.

Let $T_{\psi_1 \dots \psi_q \omega_1 \dots \omega_s}^{\varphi_1 \dots \varphi_p \lambda_1 \dots \lambda_u}$ be a given tensor in L_n . We shall assume that it is neither symmetric nor skew symmetric in $\lambda_1 \dots \lambda_u$, $\omega_1 \dots \omega_s$ and that $quv \neq 0$. Moreover, we shall use capital Greek indices Φ_p , Ψ_q , Λ_u , Ω_s instead of $\varphi_1 \dots \varphi_p$, $\psi_1 \dots \psi_q$, $\lambda_1 \dots \lambda_u$, $\omega_1 \dots \omega_s$, so that we shall write $T_{\Psi_q \Omega_s}^{\Phi_p \Lambda_u}$ instead of $T_{\psi_1 \dots \psi_q \omega_1 \dots \omega_s}^{\varphi_1 \dots \varphi_p \lambda_1 \dots \lambda_u}$. The s^{th} covariant derivative $\nabla_{\xi_s} \dots \nabla_{\xi_1} T_{\Psi_q \Omega_s}^{\Phi_p \Lambda_u}$ will be denoted by $T_{\Psi_{q+s} \Omega_s}^{\Phi_p \Lambda_u}$ and, in particular, $T_{\Psi_{q+s} \Omega_s}^{\Phi_p \Lambda_u} \stackrel{\text{def}}{=} T_{\Psi_q \Omega_s}^{\Phi_p \Lambda_u}$. Here this abridged notation has only a formal significance, later it will help us to refine certain basic notions.

By the symbol $\{T_{\Psi_{q+s} \Omega_s}^{\Phi_p \Lambda_u}\}$ we denote the set of all tensors $T_{\Psi_{q+s} \Omega_s}^{\Phi_p \Lambda_u}$ for $s = 0, 1, 2, \dots$. Let $U_{\Phi_p}^{\Psi_{q+s}}$ be any arbitrary tensor with $q + s$ contravariant and p covariant indices. Put

$$(1.1) \quad L_{\Omega_s}^{\Lambda_u}(U) \stackrel{\text{def}}{=} T_{\Psi_{q+s} \Omega_s}^{\Phi_p \Lambda_u} U_{\Phi_p}^{\Psi_{q+s}}$$

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**These tensors, invariant with respect to the holonomy group, will be investigated in the third paper of this series.

***Small Greek indices run over the range of the first n Roman numerals.