

The Time-Dependent Solution for a Storage Model with Poisson Input

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Summary. This paper is concerned with the time-dependent distribution function $F(z, t)$ of the content of an infinite dam, fed by a Poisson input and with a steady continuous release ceasing only when it becomes empty.

The forward Kolmogorov relation for $F(z, t)$ is a known difference-differential equation. By obtaining the probabilities of first emptiness of the dam, it is possible to find the probability $F(0, t)$ that the dam is empty after a period of time t , not necessarily for the first time.

This is finally used to solve the forward equation for $F(z, t)$ directly; the known stationary distribution of the dam content can then be derived as the limit of $F(z, t)$ when $t \rightarrow \infty$.

1. Introduction. In a recent note [1] and paper [2], the authors have discussed an infinite storage model, usually thought of as a dam, whose content $Z(t) \geq 0$ is defined in continuous time t ($0 \leq t < \infty$) by the equation

$$(1) \quad Z(t + \delta t) = Z(t) + \delta X(t) - \text{Min} \{Z(t), \delta t\}.$$

In this, $X(t)$ represents the input into the dam during time t , a Poisson process with parameter λ , such that in a small interval of time δt the quantity $\delta X(t) = 0$ or h may be added to the dam content. The distribution function (d.f.) of the input $X(t)$ is given by

$$K(x, t) = \sum_{r=0}^{\lfloor x/h \rfloor} e^{-\lambda t} \frac{(\lambda t)^r}{r!},$$

and it is clear from its Laplace-Stieltjes transform,

$$\int_{x=0}^{\infty} e^{-\theta x} dK(x, t) = \exp \{-\lambda t(1 - e^{-\theta h})\} = \exp \{-t\xi(\theta)\},$$

that inputs are additive, being independently distributed in non-overlapping

*The work was completed while this author was at the University of Western Australia.