

# *Functional Equations, Wave Propagation and Invariant Imbedding*

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## SUMMARY

It is shown that wave propagation in inhomogeneous media may be studied through the use of invariance principles similar to those used in radiative transfer