

*On the Integrability Conditions in the Problem of Motion in General Relativity**

JOHN MOFFAT

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Abstract. The problem is investigated of deriving equations of motion from the field equations of general relativity up to an arbitrarily high approximation. The recent work on this problem by INFELD & PLEBANSKI (1956), which presents a modified version of previous methods of attack, is applied to EINSTEIN's field equations of empty space using the EINSTEIN, INFELD & HOFFMANN surface integral technique. A detailed investigation is made of boundary and coordinate conditions, and it is deduced that the EINSTEIN, INFELD & HOFFMANN approximation method does not generally admit (approximate) homogeneous Dirichlet boundary conditions. As a consequence of this global limitation it is difficult at present to prove, by using this approximation method, that the EINSTEIN, INFELD & HOFFMANN equations of motion are unique.

1. Introduction. In the early formulation of the general theory of deriving equations of motion from the field equations a fundamental difficulty was met with in the use of the approximation method. A contradiction appeared between the approximate field equations to be solved and the surface integral conditions following from the structure of the field equations.

The basic idea behind the methods introduced by EINSTEIN, INFELD & HOFFMANN (1938)¹ and EINSTEIN & INFELD (1940, 1949)^{2,3} to overcome this difficulty was to generalize the original EINSTEIN equations so that the motion of the singularities became arbitrary; that is, arbitrary values were attributed to the $\xi^m(x^0)$ (ξ^m the position coordinates of the s^{th} singularity). The surface integral conditions were fulfilled identically, and the procedure for solving the $g_{\mu\nu}$ field approximately in terms of the field equations could go ahead without fundamental difficulty.

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¹A. EINSTEIN, L. INFELD & B. HOFFMANN, *Ann. Math.*, **39**, 66 (1938).

²A. EINSTEIN & L. INFELD, *Ann. Math.*, **41**, 797 (1940).

³A. EINSTEIN & L. INFELD, *Can. J. Math.*, **1**, 209 (1949).