

# *Solution of an Initial Value Problem for the Multi-Velocity Neutron Transport Equation with a Slab Geometry*

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This paper treats an initial value problem which arises in several branches of physical science, namely, the problem stemming from the velocity dependent integro-differential transport equation. Perhaps most notably, it occurs in the study of neutron behavior in reactor physics. Here it appears usually as a linear equation because of neglect of interactions of neutrons with neutrons.

The general linear velocity-dependent equation is seldom used directly. Rather, systems of equations of integro-differential type are obtained for discrete velocity groups, as described in the literature (Ref. 2, 3, 7 and 10). Then the initial value problem is redefined in terms of this system.

In the past, two approaches have found employment, at least in a practical way: reduction by approximation to a classical diffusion problem and numerical integration by forward differences. To the writer's knowledge, however, no penetrating analysis of the nature of this problem and its solution was ever made until the appearance in 1955-56 for two papers (Ref. 8 and 9) by J. LEHNER & G. M. WING. Dealing only with the one-velocity case, they proved existence and uniqueness for both the infinite slab and the sphere. For the slab, they were able to discuss completely the structure of the solution. The present paper is an attempt to extend the work of LEHNER & WING to the more general  $n$ -velocity case.

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1. Let  $N_i(x, \mu, t)$  be the density of neutrons at a point with space coordinate  $x$  moving in a direction with angle  $\theta$  with respect to the  $x$  axis, with speed in velocity group  $i$  at time  $t > 0$ . Let  $\mu = \cos \theta$ . The  $x$  axis is taken perpendicular

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