

*A Theorem on Integral Transforms with an Application to Prediction Theory**

J. CHOVER

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1. Introduction. The main theorem of this paper characterizes those continuous functions $f(t)$ on $a \leq t \leq b$ which can be represented as

$$(1) \quad f(t) = \int_a^b r(t-s) dg(s),$$

where $r(t)$ is a given kernel function of a certain class R (see Definition 1) and $g(s)$ is of bounded variation on $a \leq s \leq b$. Some discussion (Sec. 3) is also given of approximation to a solution by solutions of finite algebraic systems.

The type of integral kernel to be considered here was suggested by (and Theorem I has application to) the following problem in prediction theory. Let x_t , $-\infty < t < +\infty$, be a (say) real-valued, second-order-stationary stochastic process with $\mathbb{E}x_t \equiv 0$ and continuous covariance function $r(t) = \mathbb{E}x_0x_t$. Let $-\infty < a < b < c < +\infty$. There exists a well determined random variable y which is the optimal linear least-squares prediction of the variable x_c , given the x_s for $a \leq s \leq b$. The following question arises: When does there exist a function $g(s) = g(s; a, b, c)$ of bounded variation on $a \leq s \leq b$ such that

$$(2) \quad y = \int_a^b x_s dg(s)$$

with a suitable definition of the integral? Standard methods show that (2) is true if and only if

$$(3) \quad r(t-c) = \int_a^b r(t-s) dg(s), \quad a \leq t \leq b.$$

Now if one knows, for a given r , that the function $f(t) = r(t-c)$ is in the class of functions representable as in (1), the above question concerning prediction is answered in the affirmative.

The above prediction question was raised for the semi-infinite case,

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