

Unsteady Rectilinear Gas Flow

D. NAYLOR

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In the unsteady isentropic flow of a gas with one space dimension the characteristic variables can be obtained explicitly in terms of the gas speed u and the speed of sound a , and the general problem is reduced to the integration of a linear equation of the Euler-Poisson type. When the flow becomes non-isentropic the characteristic equations cannot be integrated in general and the problem becomes more difficult. MARTIN [1], [2] has developed a theory of such flows by taking as the new independent variables the pressure p and a stream function ψ applicable to such flows, reducing the general problem to the solution of a Monge-Ampère equation for which a special integral was constructed. In a previous paper [3] the equations of steady rotational plane flow of a gas were investigated by employing the stream function of CROCCO, and, in the case of generalised simple waves, considerable simplifications resulted. The object of the present paper is to study the effect on the equations of unsteady non-isentropic rectilinear gas flow of the use of a modified stream function analogous to CROCCO's stream function. This function is defined by equations (9) in terms of the gas speed u and the acoustic speed a , and remains constant for each fluid element during the motion. The method consists of applying certain transformations to a Lagrangian equation for $x(\psi, t)$, the position at time t of the fluid element ψ . The general problem is reduced to that of a quasi-linear partial differential equation homogeneous in the second order derivatives. For suitably prescribed distributions of entropy this equation possesses special integrals, one of which is equivalent to MARTIN's exact solution [5]. Other integrals are obtained by concentrating on degenerate solutions analogous to simple waves.

1. General Flow Equations. The equations of one-dimensional inviscid flow of a perfect gas are

$$(1) \quad \rho_t + (\rho u)_x = 0,$$

$$(2) \quad \rho(u_t + uu_x) + p_x = 0,$$

$$(3) \quad S_t + uS_x = 0,$$

$$(4) \quad S = C_v \log (p\rho^{-\gamma}).$$