

Positive Real Matrices

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1. Introduction. This paper is concerned with real rational functions of the complex variable $s \equiv \sigma + i\omega$ and symmetric matrices, each of whose elements is a real rational function. More specifically, we consider positive real (p.r.) functions and positive real (p.r.) matrices.

A rational function $K(s)$ is said to be p.r. if and only if:

- (1) $K(s)$ is real for s real.
- (2) $\operatorname{Re} [K(s)] \geq 0$ for $\operatorname{Re} [s] \geq 0$.

A symmetric n^{th} -order matrix of real rational functions $[a_{jk}(s)]$ is said to be a p.r. matrix if and only if its associated quadratic form

$$\sum_{j,k=1}^n a_{jk}(s)x_jx_k$$

is a p.r. function for all finite values of the real variables x_1, x_2, \dots, x_n .

The importance of the p.r. concept derives mainly from the following two facts:

- (1) A necessary and sufficient condition for a real rational function to be realizable as the driving-point impedance of a one-port network is that it be a p.r. function [1, 2].
- (2) A necessary and sufficient condition for a symmetric n^{th} -order matrix of real rational functions to be realizable as the open-circuit impedance matrix of an n -port network is that it be a p.r. matrix [3].

Some applications of the p.r. concept to networks are discussed in another paper [4] by the authors. The purpose of the present paper is to prove the previously given sets of necessary and sufficient conditions for a real rational function and a symmetric matrix of real rational functions to be p.r. The new theorems cited and proved are Theorems 1, 3, 5, 7, 9, and 10. It is believed that these new theorems are more suitable for practical testing procedures than the theorems previously given in the literature. For completeness the previously known theorems on p.r. functions and matrices are given with each new theorem.†

†An alternative characterization of *p.r.* matrices, which is equivalent to a synthesis procedure, is given by R. J. DUFFIN in his paper *Elementary Operations which Generate Network Matrices*, *Proc. Am. Math. Soc.*, 6, pp. 335–339, 1955.