

The Holonomy Group

III. Metrisable Spaces*

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1. Description of the problems. This paper is a direct continuation of [1] and [2].† It is based on the results (and symbolism) of [2]. Whenever it is necessary, these results will be quoted in the text.

Let us denote by $L_{\omega\mu\lambda}{}^\nu$ either the curvature tensor $R_{\omega\mu\lambda}{}^\nu$ of a volume preserving L_n or the curvature tensor $K_{\omega\mu\lambda}{}^\nu$ of a Riemann space V_n . In the first case $L_{\omega\mu\lambda}{}^\nu$ must satisfy the identities

$$(1.1) \quad \begin{aligned} (a) \quad L_{(\omega\mu)\lambda}{}^\nu &= 0, & (d) \quad L_{[\omega\mu\lambda]}{}^\nu &= 0, \\ (c) \quad L_{\omega\mu\lambda}{}^\lambda &= 0, & (d) \quad \nabla_{[\xi} L_{\omega\mu]\lambda}{}^\nu &= 0. \end{aligned}$$

In the second case it must satisfy the identities (1.1a, b, c, d) and

$$(1.1) \quad \begin{aligned} (e) \quad L_{\omega\mu(\lambda\nu)} &= 0, \\ (f) \quad L_{\omega\mu\lambda\nu} &= L_{\lambda\nu\omega\mu}. \end{aligned}$$

Denote by $g_{\lambda\nu}$ a symmetric tensor of rank n and by $g^{\lambda\nu}$ its inverse,

$$(1.2) \quad g^{\lambda\nu} g_{\lambda\mu} = \delta_\mu^\nu,$$

and consider the following system of equations:

$$(1.3) \quad \begin{aligned} L_{\omega\mu\lambda}{}^\nu &= \partial_{[\mu} \{ g^{\nu\alpha} (\partial_{\omega]} g_{\alpha\lambda} + \partial_{[\lambda]} g_{\omega]\alpha} - \partial_{[\alpha]} g_{\omega]\lambda} \} + \frac{1}{2} g^{\nu\beta} (\partial_\alpha g_{\beta[\mu} \\ &\quad + \partial_{[\mu} g_{\alpha\beta]} - \partial_\beta g_{\alpha[\mu}]) g^{\alpha\delta} (\partial_{\omega]} g_{\lambda\delta} + \partial_{[\lambda]} g_{\omega]\delta} - \partial_{[\delta]} g_{\omega]\lambda}). \end{aligned}$$

If $g_{\lambda\mu}$ is not known and $L_{\omega\mu\lambda}{}^\nu$ is given, (1.3) represents a system of differential equations for $g_{\lambda\mu}$. Then one of our problems may be formulated as follows:

1(a). One has to find necessary and sufficient conditions for (1.3) to admit at least one solution $g_{\lambda\nu}$.

1(b). If these conditions are satisfied, one has to find the explicit form of at least one solution $g_{\lambda\nu}$.

1(c). If one solution is given, one has to find the explicit form of all solutions.

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†These papers will be denoted by I and II.