

The Non-isentropic Perturbation of an Arbitrary Simple Wave

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1. Introduction. In a recent paper [1], a method was presented for discussing the perturbation of a given one-dimensional, non-steady flow, assumed isentropic, of an inviscid, ideal compressible gas. It was not assumed that the perturbed flow was isentropic. Explicit solutions were obtained for a basic flow which was uniform or a centered simple wave, but for an arbitrary simple wave only if the perturbed flow was isentropic. In the present paper, it is shown that the non-isentropic perturbation of an arbitrary simple wave can be solved by quadratures.

The equation of state is

$$P = \exp \left\{ \frac{s - s^*}{c_v} \right\} \rho^\gamma$$

and the equations which characterize the flow are

$$\begin{aligned} c_t + uc_x + \frac{1}{2}(\gamma - 1)cu_x &= 0, \\ u_t + uu_x + \frac{2cc_x}{\gamma - 1} - \frac{c^2 s_x}{c_v \gamma (\gamma - 1)} &= 0, \\ s_t + us_x &= 0, \end{aligned}$$

where P is the pressure, ρ the density, s the specific entropy, s^* the specific entropy at some reference state, γ the ratio of the specific heats at constant pressure c_p and at constant volume c_v , u the particle velocity, c the local speed of sound and the subscripts denote partial derivatives.

A formal linearization in the neighborhood of a known isentropic solution, denoted by the subscript zero, gives the following system of linear equations which govern the first approximation to the disturbed flow:

$$(1) \quad c_{1t} + u_0 c_{1x} + \frac{1}{2}(\gamma - 1)c_0 u_{1x} + u_1 c_{0x} + \frac{1}{2}(\gamma - 1)u_{0x} c_1 = 0,$$

$$(2) \quad \frac{1}{2}(\gamma - 1)u_{1t} + \frac{1}{2}(\gamma - 1)u_0 u_{1x} + c_0 c_{1x} + \frac{1}{2}(\gamma - 1)u_{0x} u_1 + c_1 c_{0x} = \frac{c_0^2 s_{1x}}{2c_v \gamma},$$