

*Diffraction by an Elliptic Cylinder**

BERTRAM R. LEVY

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1. Introduction. The construction of the field diffracted by a cylinder of arbitrary convex cross section has been carried out by J. B. KELLER [1], who found that by introducing a new class of diffracted rays a "geometrical optics" for diffracted fields could be developed. In all cases which have been previously studied it has been found that the field constructed by the geometrical theory of diffraction agrees with an asymptotic expansion of the exact solution of the corresponding boundary value problem. In this paper we find the asymptotic expansion of the field diffracted by an elliptic cylinder and show that it, too, is in agreement with the geometrical theory.

In Section 2 we formulate the boundary value problem for the reduced wave equation corresponding to a cylindrical wave incident upon an acoustically soft or acoustically hard elliptic cylinder. The problem is then solved by the usual separation of variables technique, and an alternative representation for the field is presented. By using the asymptotic expansions for the special functions occurring in the solution the asymptotic expansion of the diffracted field is found in Section 3. In Section 4 the diffracted field is constructed geometrically and this construction is shown to be in agreement with the asymptotic expansion of the exact solution.

In the first appendix some identities which are necessary to effect the comparison of the geometric construction and the asymptotic expansion of the exact solution are presented. In the second appendix the asymptotic expansions used in Section 3 are derived. The third appendix contains a rigorous justification of the alternative representation of the field given in Section 2.

2. The exact solution. Consider an elliptic cylinder with its axis coincident with the z axis of an (x, y, z) rectangular coordinate system. Let the interfocal distance of the generating ellipse be $2h$ and let us introduce elliptic coordinates in the (x, y) plane by

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