

# *Solution of the Dirichlet Problem for the Ellipse by Interpolating Harmonic Polynomials\**

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Since harmonic polynomials are known to possess important properties with reference to approximation to harmonic functions [WALSH, 3, 4], the possibility suggests itself [WALSH, 4, p. 517] that a sequence of such polynomials might be defined by coincidence with the values of a given function  $U(x, y)$  in suitably chosen points on the boundary of a given region  $D$ , and then might converge in  $D$  to the corresponding solution of the Dirichlet problem. The present writer has indicated [3, 4] that if  $D$  is, in particular, a circular disc, the Jackson-Bernstein-Montel-de la Vallée Poussin theory of trigonometric interpolation and approximation applies, and various results on such a solution of the Dirichlet problem are immediate.

Recently J. H. CURTISS [1] has raised anew the question of interpolation by harmonic polynomials, and he points out the analogy to interpolation by polynomials in the complex variable. He conjectures that satisfactory points of interpolation may be (for instance) equally spaced on the boundary of  $D$  with respect to the conjugate of Green's function for the complement of  $D$ . The object of the present note is to confirm this conjecture for the case that  $D$  is the interior of an ellipse, by computation as trigonometric polynomials of the harmonic polynomials involved.

**Theorem.** *Let the boundary  $B$  of a bounded region  $D$  in the  $(u, v)$ -plane be an ellipse, and let the function  $U(u, v)$  be defined and satisfy a Lipschitz condition of some positive order on  $B$  with respect to arc length. Let the harmonic polynomial  $p_n(u, v)$  of degree not greater than  $n$  coincide with  $U(u, v)$  in  $2n + 1$  points equally spaced on  $B$  with respect to the conjugate of Green's function for the exterior of  $D + B$  with pole at infinity. Then the sequence  $p_n(u, v)$  converges to  $U(u, v)$  uniformly on  $B$ , hence converges uniformly in  $D + B$  to the corresponding solution of the Dirichlet problem.*

In an arbitrary simply connected region of the  $w (= u + iv)$ -plane a complete

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