

*The Einstein Tensor in Orthogonal Coordinates in n Dimensions**

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1. Introduction. In an investigation connected with the theory of relativity I needed to know the explicit forms of the components of the Einstein tensor in an orthogonal coordinate system in an n -dimensional Riemannian space of arbitrary signature.

DINGLE [1] has given expressions for the individual components of the tensor for space-time when the line element is written in the form

$$(1) \quad ds^2 = D (dx^4)^2 - A (dx^1)^2 - B (dx^2)^2 - C (dx^3)^2.$$

By using (1), DINGLE was able to present the components of the Einstein tensor in forms well suited for specific applications, but his formulas do not immediately show the compact pattern of the components as a whole. To infer the pattern for the four-dimensional case from DINGLE's results is, of course, easy. One must replace A, B, C, D by their more familiar symbols as components of the metrical tensor, making allowance for minuses arising from the fact that $g_{11} = -A, g_{22} = -B, g_{33} = -C$, but $g_{44} = +D$.

DINGLE does not indicate his mode of calculation. If one works throughout in terms of A, B, C, D , one loses sight of the underlying pattern right from the start and is inclined to perform the calculation by brute force. Since I needed the general pattern of the components of the Einstein tensor in n dimensions, I decided to perform the calculation *ab initio*, keeping the pattern in sight by working throughout in terms of the components of the metrical tensor in the form g_{ab} rather than in terms of symbols like A, B, C, D, \dots .

The task began promisingly, but soon became so complicated that all elegance was lost and the chance of making a computational error exceeded prudent levels. After several attempts to force the calculation through, each yielding a different result, I discovered a simple manoeuvre that reduced the problem to one of surprising computational simplicity. Since the details of the method and the compact form of the results may be of interest, I present them here.

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