

# *Partial Differential Equations with More Than Two Independent Variables in the Complex Domain\**

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**Introduction.** In this paper we shall be primarily concerned with linear partial differential equations of the second order whose coefficients are analytic. We shall be interested in properties of the solutions of such equations which can be studied by means of an analytic continuation into the domain of complex values of the independent variables. The case of two independent variables has been thoroughly investigated from this point of view, and our contribution here will therefore be to problems involving three or more variables. The significance of such a study lies in the hope it offers of extending to several variables some of the theories for partial differential equations in two variables which depend on conformal mapping or on the introduction of characteristic coordinates.

The possibility of mapping an arbitrary two-dimensional surface conformally onto a plane permits us to reduce the general linear elliptic partial differential equation of the second order in two independent variables to a canonical form in which the highest derivatives appear as a Laplacian. This result is connected with the reduction to canonical form of a hyperbolic equation in two variables by means of the introduction of characteristic coordinates. The two processes are, indeed, equivalent for an equation with analytic coefficients, and the relation between them is derived through extension of the solution into the complex domain. For more independent variables a simple reduction to canonical form is not feasible, but it is not unnatural to expect that extension into the complex domain might nevertheless yield a successful analysis that could compensate in part for this deficiency. Such is the case, for example, with HADAMARD'S construction [7] of the fundamental solution of a second order equation in several variables, which he motivates by identifying the Riemann function as the coefficient of the logarithmic factor in the fundamental solution of an equation in two variables, and which he then performs in the complex domain.

One of the principal theorems about elliptic partial differential equations with analytic coefficients is that every solution must be analytic. For linear or non-

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