

Limits of Convolution Sequences of Measures on a Compact Topological Semigroup

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Introduction. Limit properties of the convolution sequence of a regular measure on a compact topological semigroup are examined in this paper. Similar questions, as they arise in the case of a compact group, were examined by KAWADA & ITO [4]. Recently BELLMAN [1] and GRENANDER [3] considered special limit theorems for products of independent identically distributed random operators. Such problems are closely related to those in this paper. It should be noted that similar questions arise when considering the structure of stationary stochastic processes [7]. Various results on compact semigroups are used in characterizing the class of limit measures [5], [6], [8].

Basic Concepts. Let S be a compact Hausdorff space. Take \mathfrak{B} as the Borel field generated by the open sets of the topology. Further, let S be a topological semigroup. This means that there is a product for pairs of elements of S and that the product operation is continuous with respect to the given topology (see [5], [6]). Let $\mathfrak{B} \times \mathfrak{B}$ be the Borel field in the product space $S \times S$ generated by the Borel field \mathfrak{B} in the component spaces. The set

$$(1) \quad A_B = \{(x, y) \mid xy \in B\} \in \mathfrak{B} \times \mathfrak{B}$$

if $B \in \mathfrak{B}$ since it is true for open sets B . We can define the product measure $\nu \times \mu$ on $\mathfrak{B} \times \mathfrak{B}$ in the usual manner [2] and the convolution $\nu * \mu$ of two measures as

$$(2) \quad \nu * \mu(B) = \int c_B(xy) d(\nu \times \mu)((x, y))$$

for $B \in \mathfrak{B}$, where c_B is the characteristic function of B .

In Lemmas 1 and 2 some simple properties of a measure on a topological space are obtained. No use is made of the semigroup property.

Lemma 1. Let ν be a measure on a Borel field \mathfrak{G} of a topological space. Let \mathfrak{F} be the collection of sets $E \in \mathfrak{B}$ such that for any $\epsilon > 0$ there are open and closed sets

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