

# *Free Boundary Problems for Parabolic Equations III. Dissolution of a Gas Bubble in Liquid*

AVNER FRIEDMAN

*Communicated by H. LEWY*

## **Introduction.**

We shall refer to [2] and [3] as Part I and Part II respectively. In Part I we considered the problem of melting of solids, the free boundary (solid-liquid line) being a monotone function of time. In Part II we considered the development of one liquid drop surrounded by totally supersaturated or totally undersaturated vapor of its own substance. In this Part III we study the behavior of one gas bubble in liquid (not necessarily of the same substance) in which some of the same gas is dissolved. There are essentially two different situations: (i) the bubble decreases and (ii) the bubble grows. It is proved that (i) takes place even in case the solution of the gas in the liquid is supersaturated, provided the initial radius of the bubble is sufficiently small (depending on the amount of supersaturation). Thus even supersaturated solutions are stable in the sense that bubbles cannot grow in them if they are too small initially. This mathematical fact was first noticed by J. B. KELLER in an unpublished manuscript [5].

In this paper we prove the existence and uniqueness of the solution and also study some of its properties. Making an appropriate transformation, the problem is reduced to the study of the system (1.16)–(1.20) where  $s(t)$  is the radius of the bubble and  $u(x, t)$ ,  $\varphi(x)$  are related in a simple manner to the density at time  $t$  and the initial density of the dissolved gas, respectively. This system in case  $m = n = 0$  is equivalent, in form, to the system of equations for the drop (in Part II, §1) provided we define, in Part II, §1,  $\bar{z} = z - 1$  and then define  $u = x\bar{z}$ . We further notice that in the case of the drop we have (when we define  $u = x\bar{z}$ )  $\varphi \leq 0$  and  $\alpha \geq 0$ , whereas in the present case of the bubble we have  $\varphi \leq 0$ ,  $\alpha > 0$  or  $\varphi \geq 0$ ,  $\alpha > 0$ . This difference, however, is of no interest since we may change the sign of both  $\varphi$  and  $\alpha$  at the same time.

An important difference between the case of the liquid drop and the case of the gas bubble is that (in physics) in the first case  $\alpha$  is of the order of magni-

---

Prepared under ONR Contract Nonr-222(37) (NR 041 157) with the University of California in Berkeley.