

On Nonlinear Ordinary Differential Equations Containing a Parameter

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0. Introduction. We shall present a method of solving a system of differential equations of the type

$$(A) \quad \epsilon^\sigma \frac{dy}{dx} = f(x, y, \epsilon),$$

where σ , ϵ , x and y are a positive integer, a complex parameter, a complex variable and an n -dimensional vector, respectively. One norm of a vector y is given by

$$\|y\| = \max \{|y_j|; j = 1, 2, \dots, n\},$$

where y_j , $j = 1, \dots, n$, are the components of y . Each component of the n -dimensional vector f is assumed to be holomorphic in all its arguments in a region defined by

$$(B) \quad |x| \leq \delta_0, \quad \|y\| \leq \delta'_0, \quad 0 < |\epsilon| \leq \rho_0, \quad |\arg \epsilon| \leq \alpha_0,$$

where δ_0 , δ'_0 , ρ_0 and α_0 are all positive constants. In addition, we shall suppose that f admits for

$$(C) \quad |x| \leq \delta_0, \quad \|y\| \leq \delta'_0$$

a uniformly asymptotic expansion

$$(D) \quad f(x, y, \epsilon) \simeq \sum_{k=0}^{\infty} \hat{f}_k(x, y) \epsilon^k$$

as ϵ tends to zero through the region

$$(E) \quad 0 < |\epsilon| \leq \rho_0, \quad |\arg \epsilon| \leq \alpha_0.$$

The coefficients \hat{f}_k are assumed to be holomorphic in (C).

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