

*Analytic Continuation of Random Series*¹

ALEXANDRA IONESCU TULCEA

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The present work is divided in two parts. In the first part we study some analytic aspects of the ergodic theory of random series. To be more precise, we consider a non-atomic measure space (X, \mathfrak{B}, μ) of finite total measure and an ergodic transformation τ on X and for every $f \in L^1(X, \mathfrak{B}, \mu)$ we define the power series

$$F(f; x, z) = \sum_{n=0}^{\infty} f(\tau^n(x))z^n, \quad x \in X, \quad |z| < 1.$$

We are concerned especially with the problem of analytic continuation of $F(f; x, z)$ for almost every $x \in X$. No separability assumptions on the underlying measure space or invertibility assumptions on the ergodic transformation τ are made (except when explicitly stated otherwise).

In the second part of the work we consider an abstract infinite-dimensional Hilbert space \mathfrak{H} and the set \mathfrak{U} of all unitary operators on \mathfrak{H} . We study various equivalent aspects of the fact that the spectrum of a given operator $U \in \mathfrak{U}$ coincides with the whole unit circle.

We shall denote by C the unit circle $\{z \mid |z| = 1\}$.

PART I

1. Preliminaries. Let (X, \mathfrak{B}, μ) be a measure space. In what follows we shall assume that:

1° $\mu(X) = 1$.

2° The measure space (X, \mathfrak{B}, μ) is non-atomic.

Let τ be a measurable, measure-preserving mapping of X into itself and let $f \in L^p(X, \mathfrak{B}, \mu)$ ($1 \leq p \leq \infty$). Define the function

$$(1) \quad F(f; x, z) = \sum_{n=0}^{\infty} f(\tau^n(x))z^n, \quad x \in X, \quad |z| < 1.$$

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