

# *Invariant Imbedding, Random Walk and Scattering-II. Discrete Versions*

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**1. Introduction.** In the first paper of this series, [1], we sketched the application of the techniques of invariant imbedding to various random walk processes and questions of scattering theory. In this paper, we wish to fill in the details and to show how the same ideas enable us to treat multi-dimensional, time-dependent, and energy-dependent processes.

A remarkable formal equivalence holds in all of these cases, with the result that the same equations—in different variables naturally—occur over and over.

As in other applications of invariant imbedding, we obtain both new analytic and new computational approaches. In some cases, we can transform problems of formidable proportions along classical lines to reasonable size.