Intrinsic Form of the Characteristic Relations for a Perfect Compressible Fluid in General Relativity and Non-Steady Newtonian Mechanics*

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1. Introduction. By use of the relation between the vector along a world-line, the normal to a characteristic surface, and a vector along the bicharacteristic corresponding to this normal, the equations of motion and continuity of an isentropic, perfect, non-self-gravitating fluid in general relativity [1] are obtained in intrinsic form. That is, these equations are written in a form involving the sound speed a, the speed of light c, the derivatives of the sound speed along the normal and bicharacteristic directions, and the derivatives of the normal and bicharacteristic. By introducing two unit vectors, both of which are space-like and both of which are orthogonal to the normal and bicharacteristic at each point, the equations of motion and continuity can be replaced by four equations which involve the above noted derivatives of the sound speed as well as its derivatives in the directions of the two unit vectors, the curvatures of the curves along the normal, bicharacteristic, and the remaining two directions, and the curvature of the characteristic surfaces. These are the intrinsic forms of the equations of motion and continuity.

The remainder of the paper, except Section 6, is concerned with the case when the characteristic surfaces form a family of ∞^1 parallel hyperplanes and one of the two unit vectors introduced earlier generates a family of ∞^3 parallel straight lines. The existence of the hyperplanes and straight lines implies that the space considered is one of special relativity. Further, the curvatures of the above congruences can now be expressed in terms of some of the partial derivatives of the metric tensor. These last derivatives can in turn be expressed in terms of the derivatives of one function ψ . Thus the equations of motion

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