

# *The Problem of Evolution in General Relativity*

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**Summary.** It is shown that the problem of evolution in general relativity need have no unique solution determined by continuity alone if matter is present as a perfect fluid. This is demonstrated by two examples. One of these is a model universe, in agreement with the cosmological observations, which arises by a smooth transition from flat space-time.

**1. Introduction.** Existence theorems for the problem of evolution in general relativity have been given by FOURÈS-BRUHAT [1]. Her results are, roughly, the following. Corresponding to given Cauchy data on a space-like hypersurface the field equations

$$(1.1) \quad R_{ik} = 0$$

have one and only one solution. The theorem is subject to the Cauchy data and the metric tensor satisfying certain differentiability conditions. A similar theorem applies for the field equations

$$(1.2) \quad R_{ik} - \frac{1}{2}g_{ik}R = -8\pi\rho u_i u_k ,$$

that is to say, if "pure matter" is present.

A different way of stating these results is as follows. Given for  $t \leq 0$  a metric tensor  $g_{ik}$  satisfying (1.1) (or (1.2)), then  $g_{ik}$  has a unique continuation satisfying (1.1) (or (1.2)) for  $t > 0$ , provided appropriate continuity conditions are satisfied on  $t = 0$ .

It will be proved in this paper that these results cannot in general be extended to the case where matter is present in the form of perfect fluid. The method will be to show by an example (in §2) that if the energy tensor corresponds to a perfect fluid, one can obtain for  $t > 0$  a metric (with non-zero pressure and density) continuous at  $t = 0$  with flat space-time. Flat space-time is a solution corresponding to a perfect fluid with zero pressure and density, and one continuation of it is, of course, flat space-time itself. It therefore follows that the problem of evolution is not uniquely determined by continuity alone in this case.