

# On Yilmaz' New Approach to General Relativity

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**I. The Theorem of Yilmaz.** YILMAZ<sup>1</sup> has presented a theory of gravitation in which he regards the metrical tensor  $g_{\mu\nu}$  as a functional of a scalar function  $\varphi$ . His basic equations are

$$(1) \quad R_{\mu}^{\nu} - \frac{1}{2} \delta_{\mu}^{\nu} R = 8\pi T_{\mu}^{\nu}$$

and

$$(2) \quad g^{\mu\nu} \varphi_{,\mu\nu} \equiv (-g)^{-\frac{1}{2}} \{ g^{\mu\nu} (-g)^{\frac{1}{2}} \varphi_{/\mu} \}_{/\nu} = 0$$

where the comma denotes the covariant derivative with respect to the metric  $g_{\mu\nu}$ , the solidus denotes the partial derivative, and the tensor  $T_{\mu}^{\nu}$  is defined by

$$(3) \quad T_{\mu}^{\nu} = \frac{1}{8\pi} (2g^{\nu\lambda} \varphi_{/\mu} \varphi_{/\lambda} - \delta_{\mu}^{\nu} g^{\lambda\tau} \varphi_{/\lambda} \varphi_{/\tau}),$$

this being the usual energy-momentum tensor for the scalar field  $\varphi/(8\pi)^{\frac{1}{2}}$ .

He exhibits a static field based on the following remarkable theorem that he discovered.

Take the line element to have the special form

$$(4) \quad ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} = e^{-2\varphi} dt^2 - e^{2\varphi} (dx^2 + dy^2 + dz^2)$$

with  $\varphi$  independent of time, and note that, for this line element, the field equation (2) reduces to the "flat" Newtonian equation

$$(5) \quad \nabla^2 \varphi \equiv \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial z^2} = 0.$$

Now compute the Einstein tensor  $R_{\mu}^{\nu} - \frac{1}{2} \delta_{\mu}^{\nu} R$  for the metric in (4), and also compute the tensor  $T_{\mu}^{\nu}$  defined in (3) using this metric. Then the two expressions will be identical—except for one discrepancy in the 4-4 components. We have, in fact, for this special line element,

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<sup>1</sup>H. YILMAZ, *Phys. Rev.* **111**, 1417 (1958).