

# The Holonomy Group

## IV. The General $L_n$ with Symmetric Connection\*

VÁCLAV HLAVATÝ

**1. Introduction.** This paper is a continuation of the papers [1], [2], [3] of the same title.† It uses the same symbolism and some results of these papers. In particular, we shall again denote by  $L_n$  an  $n$ -dimensional space endowed with a symmetric connection  $\Gamma_{\lambda}{}^{\nu}{}_{\mu} = \Gamma_{\mu}{}^{\nu}{}_{\lambda}$  and by  $R_{\omega\mu\lambda}{}^{\nu}$  its curvature tensor and put

$$R_{\omega\mu} \stackrel{\text{def}}{=} R_{\omega\mu\alpha}{}^{\alpha}.$$

The connection  $\Gamma_{\lambda}{}^{\nu}{}_{\mu}$  (or the space  $L_n$ ) is termed volume preserving if

$$R_{\omega\mu} = 0.$$

This condition characterized [III], where we dealt with a Riemann space  $V_n$  (*i.e.* an  $L_n$  with covariant constant quadratic tensor  $g_{\lambda\mu} = g_{\mu\lambda}$  of rank  $n$ ). In this paper we shall deal exclusively *with an  $L_n$  which is not volume preserving*:

$$(1.1) \quad R_{\omega\mu} \neq 0.$$

\* \* \*

The paper contains three parts. The first part serves as an introduction to the second part. We define an  $L_n$  in a manner which is slightly different from the usual definition. This enables us to stress the condition (1.1) by means of a vector  $Q_{\mu}$  and its alternating derivative  $\partial_{[\omega} Q_{\mu]}$ . The second part, comprising sections 5–12, deals with the holonomy group  $G_l(R)$  of an  $L_n$  which enables us to state in a concise form the complete set of integrability conditions of certain systems of linear partial differential equations. The condition (1.1) is very important for the structure of the holonomy group. It turns out that there are two different types of holonomy groups. Both contain at least two subgroups, one of them being always normal (*i.e.* invariant with respect to the transformations of the adjoint group). In the second type of holonomy group the remaining

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†These papers will be denoted by [I], [II], [III].