

On Quasi-Linear Parabolic Equations of the Second Order II

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Introduction. Let D denote a bounded $(n + 1)$ -dimensional domain of real variables $(x, t) = (x_1, \dots, x_n, t)$. We assume that D is bounded by two hyperplanes $t = 0$ and $t = t_0 > 0$, and by a surface S (S is a closed set) lying between the hyperplanes. Denote by \bar{D} the closure of D , by B the base of D (that is, $B = \bar{D} \cap \{t = 0\}$), by D_τ the domain $D \cap \{0 < t < \tau\}$, by S_τ the intersection $S \cap \{0 \leq t \leq \tau\}$, by \bar{D}_τ the closure of D_τ , and, finally, denote by ∂D_τ (∂D) the set $S_\tau \cup B$ ($S \cup B$).

Consider the linear parabolic operator

$$(0.1) \quad Lu \equiv \sum_{i,j=1}^n a_{ij}(x, t) \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^n b_i(x, t) \frac{\partial u}{\partial x_i} + c(x, t)u - \frac{\partial u}{\partial t}$$

with coefficients defined in \bar{D} . In [2] we proved existence and uniqueness theorems for the first mixed boundary problem:

$$(0.2) \quad Lu = f(x, t, u, \nabla u) \quad \text{for } (x, t) \text{ in } D,$$

$$(0.3) \quad u = h(x, t) \quad \text{for } (x, t) \text{ on } \partial D$$

where $\nabla u = (u_{x_1}, \dots, u_{x_n})$ and where f is defined in the domain

$$(0.4) \quad (x, t) \in \bar{D}, \quad -\infty < u < \infty, \quad -\infty < u_{x_i} < \infty \quad (i = 1, \dots, n).$$

The proof of the existence of solutions depended upon an *a priori* $(1 + \delta)$ -estimate which was derived in the same paper. In deriving that estimate, however, we employed Green's function and quoted results which (as we noticed later) are not known. So that proof is incomplete.

In the present paper, we not only complete the proof of the $(1 + \delta)$ -estimate, but we also derive it under much weaker assumptions than in [2]. As a consequence, we obtain existence theorems under very weak differentiability assump-

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