

# *Non-uniqueness in Cauchy's Problem for Differential Equations of Elliptic Type*<sup>1</sup>

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It has been known since 1901 [2] that Cauchy's problem for linear partial differential equations with analytic coefficients has at most one (smooth) solution, provided that the data are given on a non-characteristic manifold. This theorem was extended [1] to the case of Hölder continuously differentiable coefficients under the additional assumption that the characteristics are non-multiple. The example in [3] shows that multiplicity of real characteristics may cause non-uniqueness even in the case of coefficients in  $C^\infty$ .

The problem arises whether the multiplicity of essentially complex characteristics can be admitted in the uniqueness theorem. L. HÖRMANDER proved a uniqueness theorem for equations having constant leading coefficients, with at most double multiplicity of essentially complex characteristics.<sup>2</sup>

In this paper we show that the answer to this problem is in general negative for coefficients in  $C^m$  ( $m$  finite). The precise statement is as follows.

**Theorem.** *There exists a linear system of partial differential equations of elliptic type for which the Cauchy problem possesses two distinct solutions of class  $C^\infty$ . The leading coefficients are constant and the remaining ones are of class  $C^m$  ( $0 \leq m < \infty$ ). The (imaginary) characteristics are  $(m + 3)$ -fold. The system consists of  $2(m + 3)$  complex equations in two independent (real) variables; it corresponds to  $4(m + 3)$  real equations defined on the whole plane.*

(The coefficients and solutions are analytic in  $x$ ).

The system is ( $p = m + 3$ )

$$\begin{aligned}u_i^j - iu_x^j + iu_x^{j+1} &= f^j(t, x)v^1 & (j = 1, \dots, p - 1), \\u_i^p - iu_x^p &= \lambda(t)u^1 + f^p(t, x)v^1 \\v_i^j - 2iv_x^j + 2iv_x^{j+1} &= g^j(t, x)u^1 & (j = 1, \dots, p - 1), \\v_i^p - 2iv_x^p &= \mu(t)v^1 + g^p(t, x)u^1\end{aligned}$$

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<sup>2</sup> For further bibliography about uniqueness in Cauchy's problem, see [1].