

On the Boundary Behavior of a Conformal Map of the Open Unit Disk into a Riemann Surface

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Communicated by T. RADÓ

1. The study of the boundary behavior of a meromorphic function whose domain is the open unit disk has received considerable attention. The list of contributions to the field is extensive. There does not appear to be, however, a systematic investigation of the boundary behavior of a conformal map of the open unit disk into an arbitrary Riemann surface, even though the concepts introduced in the classical case of a meromorphic function are not dependent in any essential way on the fact that the image of the map under consideration lies in the extended plane. The object of the present note is to extend two well-known theorems of the theory of the boundary behavior of meromorphic functions to the case of a conformal map of the open unit disk into an arbitrary Riemann surface. The theorems in question are due respectively to COLLINGWOOD & CARTWRIGHT [3] and to CARATHÉODORY [2]. It will be seen that the methods we employ differ considerably from those of the aforementioned authors.

No attempt is made to construct a systematic theory of the boundary behavior of a conformal map of a Riemann surface in the present paper; rather, our attention will be confined to the two theorems to which reference has been made.

2. **The theorem of Collingwood and Cartwright.** We recall the basic terms involved in the theorem of COLLINGWOOD & CARTWRIGHT and its statement. Let f be a meromorphic function whose domain is $\{|z| < 1\}$. A point ζ of the unit circumference is termed a *Fatou point* of f provided that f possesses a sectorial limit at ζ . We let F denote the set of Fatou points of f . By the *range* of f at ζ , $|\zeta| = 1$, is understood the set of points w of the extended plane for which ζ is an accumulation point of $f^{-1}(\{w\})$. A point ζ of the unit circumference is termed a *Picard point* of f provided that the range of f at ζ omits at most two points of the extended plane. We denote the set of Picard points of f by P . The theorem of COLLINGWOOD & CARTWRIGHT asserts