

*A Note on the Concept of Entropy*¹

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Introduction. From a physical point of view, there is no reason why entropy should not be defined for any state of any physical system. The present paper gives a mathematical formulation that is roughly of this order of generality, which it validates by deriving certain properties. It is shown that the entropy of a mixture of states is bounded from below by the corresponding average of the entropies of the constituent states, that the entropy of a state is increased by restricting it to a subsystem, and that pure and equilibrium states tend to be characterized by their entropies. No difficult technical considerations are involved, although some of the problems that are led up to seem to involve quite non-trivial questions of technique.

As is well known, entropy appears originally in thermodynamics, plays a part in the related subject of statistical mechanics, extends appropriately to quantum mechanics, and more recently has emerged as a fundamental notion for information theory, in a form somewhat related to earlier and simpler ideas concerning statistical estimation (see notably [7] and [8]). What is common to all these areas is the existence of a physical system that may be defined through its observables (or "random variables" in probability theory) and a concern with the states of this system as defined through the expectation values of the observables. A viable theory dealing with such matters has been set up on the basis of a few quite conservative postulates in [3], but the virtually exclusive theoretical prototype for the physical systems that have arisen or are even contemplated is included in the special case in which the observables may be imbedded in an associative algebra with a related adjunction operation, which has the effect that, technically, the observables may be represented by the self-adjoint elements of an algebra of operators in Hilbert space. Actually, the analytical form of the systems that arise in quantum mechanics and probability theory is such that they have certain general properties beyond that of being represented by an algebra of operators that is closed under uniform convergence and under the adjunction operation, which is all that develops from the general phenomenological and elementary algebraic considerations

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