

On the Geometrical Significance of the Moments of Order Statistics and of Deviations of Order Statistics from the Mean in Samples from Gaussian Populations

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1. Introduction and summary. Let $\xi \equiv \xi_{r|n}$ denote the r^{th} largest observation in a random sample of n observations from a normal population with zero mean and unit variance. Then the elementary probability law of ξ is given by

$$(1) \quad n \binom{n-1}{r-1} [\Phi(\xi)]^{r-1} [1 - \Phi(\xi)]^{n-r} \phi(\xi),$$

where

$$\begin{aligned} \phi(\xi) &= (2\pi)^{-\frac{1}{2}} e^{-\frac{1}{2}\xi^2}, \\ \Phi(\xi) &= (2\pi)^{-\frac{1}{2}} \int_{-\infty}^{\xi} e^{-\frac{1}{2}\eta^2} d\eta. \end{aligned}$$

(See *e.g.* WILKS [4].)

We shall investigate the distribution of $\xi_{r|n}$, the r^{th} order statistic, and that of $\xi_{r|n} - \bar{\xi}$, the deviation of the r^{th} order statistic from the sample mean ($\bar{\xi} = \sum_{i=1}^n \xi_{i|n}/n$), from the point of view of the moments of these distributions. It will appear that the latter moments are directly related to certain *geometrical* moments of members of a class of simplices, referred to as skew-regular simplices, in $(n - 2)$ -dimensional spherical space.

Skew-regular hyperspherical simplices of dimensionality $m - 1$ are generated from a regular hyperspherical simplex as follows. Consider a regular simplex constructed on the surface of a sphere (which for convenience may be taken as

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