

*The Content of a Dam as the Supremum of an Infinitely Divisible Process**

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Summary. This paper discusses the relation between the content of a dam and the supremum of a certain infinitely divisible process.

It is first pointed out that the distribution functions of the supremum over time $0 \leq t \leq T$ of a Poisson process with shift and the content at time T of a dam with Poisson inputs and a constant release rate have identical forms. The backward Kolmogorov equation of the first process is shown to be identical with the forward equation of the second.

A closer relation between the two processes is proved to exist, and a new definition for dam processes in discrete time is given. This is generalized to dam processes in continuous time whose net inputs are from a wide class of infinitely divisible processes.

The distribution function of the total time during which such dams are empty and non-empty is found. Lastly, using a result of KENDALL (1957) in the theory of dams, a simplification of the double Laplace transform due to BAXTER & DONSKER (1957) for the distribution function of the supremum over $0 \leq t \leq T$ of certain infinitely divisible processes with positive jumps is obtained.

1. Introduction. In a recent paper, PYKE (1959a) has obtained the inversion of BAXTER & DONSKER's (1957) double Laplace transform (see Section 8) for the distribution function (d.f.)

$$(1.1) \quad \sigma(x, T) = P \left[\sup_{0 \leq t \leq T} X_t \leq x \right]$$

of the supremum of a separable centered Poisson process with shift $\{X_t : t \geq 0\}$ satisfying $X_0 = 0$, whose characteristic function is

$$(1.2) \quad E(e^{i\omega X_t}) = \exp \{-it\omega\alpha + \lambda t(e^{i\omega} - 1)\},$$

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