

# *The Characteristic System for Three-dimensional, Steady, Isentropic, Compressible, Hydromagnetic Flows*

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*Communicated by* S. CHANDRASEKHAR

**1. Introduction.** FRIEDRICHS & KRANZER [1] have derived the partial differential equation for the characteristic manifolds in hydromagnetic flows by use of the weak shock technique. In the present paper, we shall (1) study this equation in order to determine the properties of the normals to characteristic surfaces in the steady three-dimensional case and (2) determine, by use of these results, the characteristic system for steady, three-dimensional, compressible, isentropic, hydromagnetic flows.

The normal cone with vertex at any point  $P$  is studied by determining the projection onto the local plane formed by the velocity and magnetic field vectors at  $P$  of the intersection of the cone of normals and the unit sphere with center at  $P$ . The resulting locus  $L$  furnishes a geometric picture of the normal cone and is easily analyzed. It is shown that *the normal cone exists for all steady, three-dimensional, isentropic, compressible (or incompressible) hydromagnetic flows*. The locus  $L$  depends upon two positive scalars:  $M$ , the Mach number, and  $K^2$ , the ratio of the magnetic and kinetic energies. Further, *there exist at least two distinct normals  $n^\lambda$ ,  $'n^\lambda$  which lie in the plane of the distinct velocity and magnetic field vectors at each point  $P$ , and there exists one normal  $k^\lambda$  orthogonal to this plane* in any steady, three-dimensional, compressible (or incompressible) hydromagnetic flow. These normals,  $n^\lambda$ ,  $'n^\lambda$ ,  $k^\lambda$ , determine three families of characteristic surfaces which will be used in formulating the characteristic system for three-dimensional hydromagnetic flows. The intersections of these characteristic surfaces lie along  $k^\lambda$  and  $t^\lambda$ ,  $'t^\lambda$ , which are the bicharacteristics corresponding to  $n^\lambda$ ,  $'n^\lambda$ , respectively, and lie in the local plane formed by the velocity and magnetic field vectors ( $v^\lambda$  and  $H^\lambda$ , respectively).

The characteristic system consists of three types of relations. First, the relations expressing  $'n^\lambda$ ,  $n^\lambda$ ,  $t^\lambda$ ,  $'t^\lambda$  in terms of  $v^\lambda$ ,  $H^\lambda$  are determined. If *the de-*

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This work was done under a grant of the National Science Foundation (NSF-G10103), administered by the University of Michigan Research Institute.