

*Proper Time, Apparent Time, and Formal Time in the Twin Paradox**

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Introduction. In this paper we shall study in more detail some phenomena connected with the so-called twin paradox,† distinguishing strictly between the *purely geometrical aspect* of this problem and its *physical application*. Using the non-relativistic language (which is not adequate) we may describe the paradox as follows (Fig. 1). While one observer P_1 stays at o , the other observer P_2 first recedes from P_1 , till h , and then returns to o . This motion is uniform (rectilinear) and the velocity in the $\begin{cases} \text{first} \\ \text{second} \end{cases}$ phase of the experiment is $\begin{cases} v = \text{Tg } \omega \\ -v = -\text{Tg } \omega. \end{cases}$ If (P_i) ($i = 1, 2$) denotes the biological time spent by P_i during the whole experiment then

$$(A) \qquad (P_1) = (P_2) \text{Cos } \omega.$$

In this paper we shall consider only the geometrical aspect of the paradox. From the point of view of geometry the only paradoxical thing about this problem is its name. In fact, in the geometrical interpretation of (A) the symbol (P_i) stands for the proper time of P_i , *i.e.*, for the Lorentz length of the world line w_i of P_i . Then (A) is simply a consequence of the elementary statement that for the Lorentz lengths (ab) , (bc) , (ca) of the edges of a time-like triangle abc we have

$$(ab) > (bc) + (ca).$$

This is a fundamental relation in the Lorentz geometry, and there is nothing paradoxical about it.

On the other hand, there is something deeper in this problem than meets the eye on first glance. From the relativistic point of view the statement that

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† There is a recent comprehensive article describing this paradox, A. SCHILD, The clock paradox in relativity theory, *American Mathematical Monthly*, **66**, pp. 1–18, 1959.